ON THE SHAPE DERIVATIVE OF THE VOLUME INTEGRAL OPERATOR IN ELECTROMAGNETIC SCATTERING BY HOMOGENEOUS BODIES

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Abstract. We establish a complete mathematical framework for the shape derivative of the strongly singular volume integral operator that describes time-harmonic scattering by a homogeneous medium in electromagnetism. Furthermore, we use these results to show the differentiability of the solution to the electromagnetic problem with respect to variations of the shape and structure of the scatterer, and we give a characterization of the first shape derivative as a solution to a volume integral equation.

We consider the scattering of time-harmonic electromagnetic waves by a bounded penetrable obstacle occupying a domain $\Omega \subset \mathbb{R}^3$. The mathematical model is given by the Maxwell equations involving the wave number $k \in \mathbb{R}$, the electric permittivity ε and the magnetic permeability μ. We consider here that ε and μ are piecewise constant with discontinuity on the surface Γ of the scatterer Ω.
The study of the differentiability of the solution of such a scattering problem with respect to transformations of the body Ω is of large importance for solving optimal shape design and inverse problems. If Ω is sufficiently smooth, a characterization of the derivative as a solution of a new transmission problem was already obtained using a variational formulation [3] or with the help of a boundary-integral representation [1, 2].
In this talk, we describe another approach, based on volume integral operators, that provides a new characterization of the shape derivative of the solution when Ω is only Lipschitz.
We begin by considering the following volume integral equation which is, under the above assumptions, equivalent to our scattering problem (see, for example, [4]):

$$\text{Find } u \in H(\mathsf{curl}, \Omega) \text{ such that } u - \eta A_k u - \nu B_k u = u^0,$$

where u^0 is a data, η and ν are respectively the electric and the magnetic contrast, and where the integral operators A_k and B_k are given, for $x \in \Omega$ by

$$A_k u(x) = (-\nabla \operatorname{\mathsf{div}} - k^2) \int_\Omega g_k(x-y)\, u(y)\, dy, \quad B_k u(x) = \operatorname{\mathsf{curl}} \int_\Omega g_k(x-y) \operatorname{\mathsf{curl}} u(y)\, dy\,;$$

with g_k the fundamental solution of the Helmholtz equation.
We then establish some results on the shape derivative of A_k and B_k. More precisely, and under suitable regularity conditions on the diffeomorphism transforming the reference domain Ω, we show the existence and a representation of the derivative for each of the two integral operators. As a consequence, we deduce a characterization of the shape derivative of the solution to the corresponding electric or magnetic problem as a solution of a new volume integral equation. Moreover, we give an algorithm to compute the derivative.

References

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