CONTROLLABILITY OF A SEMI DISCRETIZED PARABOLIC EQUATION VIA THE MOMENT METHOD

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Abstract. We are interested in null controllability in time $T > 0$ of a semi discretized in space parabolic equation, in dimension 1. We study the case of a distributed control as well as the one of a boundary control via the moment method (see [3]) – or a variation of which – adapted to the discrete setting.

Given $\mathcal{A}^h$ a discretization of the following symetric elliptic operator : $\mathcal{A} := -\partial_x(\gamma(x)\partial_x\cdot) + q(x)\cdot$ on a discret fonctional space E^h, the problem reads as follow :

$$\begin{cases} {y^h}'(t) + \mathcal{A}^h y^h(t) = \mathcal{B}^h u^h(t), \text{ for } 0 < t \leq T \\ y^h_0(t) = 0,\, y^h_{N+1}(t) = Cv^h(t), \text{ for } 0 < t \leq T, \\ y^h(0) = y^{0,h} \in E^h. \end{cases} \tag{0.1}$$

We study the case of a distributed control ($C = 0$, $\mathcal{B}^h = 1_\omega$, $u_h(t) \in E^h$) as well as the one of a boundary control ($\mathcal{B}^h = 0$, $C = 1$, $v^h(t) \in \mathbb{R}$). In the case where $\gamma = 1$ and with a uniform discretization, we get the null boundary and distributed controllability of the control problem (0.1). Moreover we managed to prove a uniform upper bound for the discrete controls u^h and v^h. The method consists in introducing a biorthogonal family to $(e^{-\mu^h_j t})_j$ where $(\mu^h_j)_j$ describes the set of eigenvalues of the discret laplacian operator with which we concatenate the eigenvalues of the continuous laplacian operator.

This work extends the results of [4], where a similar result is proved only in the case where $\mathcal{B}^h = 0$ (boundary control problem) and $q = 0$ and $\gamma = 1$ (laplacian operator) on a uniform mesh. These authors deal with the explicit formula of the spectrum of the operator $\mathcal{A}^h$ which are not known in the general setting under study.

In the general case, we are not able to prove uniform bounds on the controls (besides, this result does not hold in dimension 2). However an alternative solution consists in driving the solution y^h at time T to an exponentially small target using uniformly bounded controls. We tackle low frequencies and high frequencies separately to this end. In particular, in 1–d, we recover the results of [2] (obtained using discret Carleman inequalities) with a more elementary approach.

Numerical simulations will enlighten the talk, based on the penalised HUM method presented in [1] in the case of distributed control and adapted to the case of boundary control.

References

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