MINIMAL PERIMETER PARTITIONS ON MANIFOLDS

BENIAMIN BOGOSEL [1] ÉDOUARD OUDET [2].

[1] *Universite Grenoble Alpes, Laboratoire Jean Kuntzmann.*
[2] *Universite Joseph Fourier, Laboratoire Jean Kuntzmann..*

ABSTRACT. We present a relaxed framework for the approximation of partitions which minimize the total perimeter under area constraints on surfaces. We present details concerning the numerical implementation and present some of the numerical results. We also give a method based on optimization on meshes which allows the precise extraction of the contours of the partitions and the computation of the optimal cost.

1. INTRODUCTION

Following the ideas presented in [3], the goal is to find numerically minimizers of the functional

$$\mathcal{F}((\omega_i)) = \sum_{i=1}^n \mathrm{Per}(\omega_i)$$

where (ω_i) is any partition of a three dimensional surface S into domains of prescribed areas $|\omega_i| = c_i$ with $\sum_{i=1}^n c_i = |S|$. Classical problems which fall into this category include partitions of the sphere into domains with equal areas. Cox and Fikkema performed numerical computations in the case of the sphere [2] using the software Evolver [1]. We propose a different numerical framework with the aid of the following relaxed version of $\mathcal{F}$:

$$\mathcal{G}_\varepsilon(u_i) = \sum_{i=1}^n \int_S \left(\varepsilon|\nabla_\tau u_i|^2 + \frac{1}{\varepsilon}u_i^2(1-u_i)^2\right) \tag{1.1}$$

where u_i are approximations of the characteristic functions of ω_i which satisfy $\sum_{i=1}^n u_i = 1$ (partitioning condition) and $\int_S u_i = c_i$ (fixed areas conditions). This approximation argument is made rigorous with the aid of a Γ-convergence theorem.

2. NUMERICAL RESULTS

We compute first the numerical minimisers of G_ε for different types of surfaces. Examples can be seen in Figure 1. Then we propose a method for the extraction of the contours and the computation of the associated optimal costs by performing an optimization on the triangulated surfaces. Some examples can be seen in Figure 2. We are able to confirm the results presented in [2] in the case of the sphere. The advantage of our method is that we can study any surface we want once we have a triangulation.

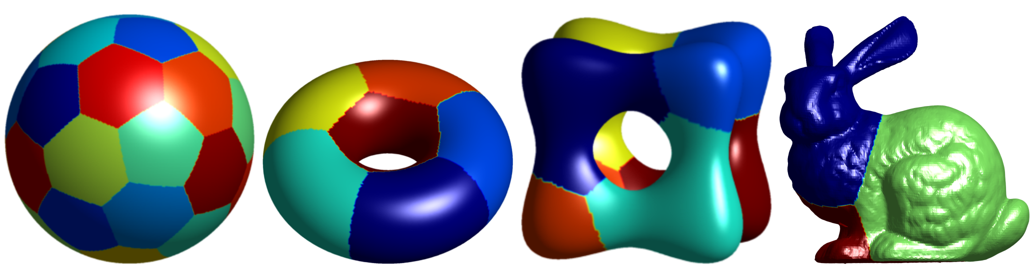

FIGURE 1. Numerical results - relaxed formulation

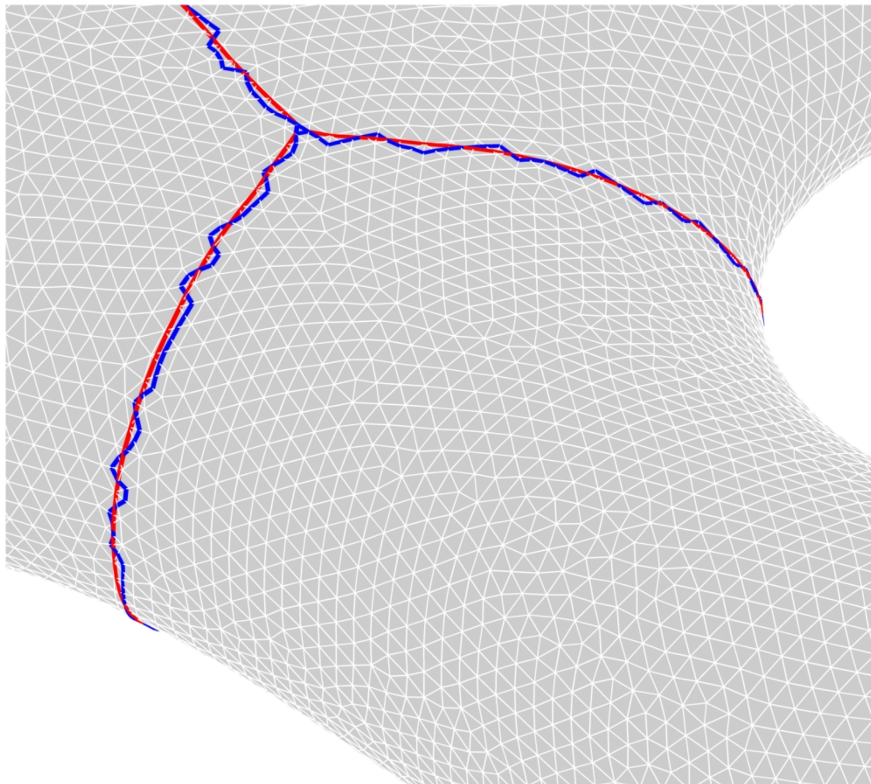

FIGURE 2. Contour extraction and optimization on meshes. Initial contour - blue vs optimized smooth contour - red.

References

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