# CAVITIES IDENTIFICATION FROM PARTIALLY OVERDETERMINED BOUNDARY DATA IN LINEAR ELASTICITY

A. BEN ABDA [1], E. JAÏEM [1], B. RJAIBI [1].

[1] *LAMSIN-ENIT, University of Tunis El Manar, BP 37, 1002, Tunis, TUNISIA.*

Abstract. A new framework for a geometric inverse problem in linear elasticity is investigated. The problem concerns the recovery of cavities from the knowledge of the displacement field and the normal component of the normal stress. We propose an identification method based on an energy gap functional combined with the topological gradient method. Numerical experiments carried out by means of a one-shot reconstruction algorithm highlight the efficiency of the proposed method to recover multiple cavities.

## 1. Introduction

Let $\Omega \subset \mathbb{R}^2$ denotes an open and bounded domain with boundary $\Upsilon$ occupied by a linear elastic material, the medium being assumed to be homogeneous and isotropic. Given the normal component of the normal stress imposed $g$ and the displacement field $f$ measured on the boundary $\Upsilon$, the geometric inverse problem consists in finding the boundary $\Gamma$ of a bounded domain $\overline{\mathcal{O}} \subset \Omega$ and the displacement field $u$ satisfying

$$\begin{cases} -\operatorname{div}\sigma(u) & = 0 & \text{in } \Omega \setminus \overline{\mathcal{O}}, \\ \sigma(u)\, n & = 0 & \text{on } \Gamma, \\ u & = f & \text{on } \Upsilon, \\ \sigma(u)\, n_\Upsilon \cdot n_\Upsilon & = g & \text{on } \Upsilon. \end{cases} \tag{1.1}$$

The geometrical inverse problem (1.1) is formulated as a topology optimization one by minimizing an energy gap functional defined by

$$\mathcal{J}(u^D, u^N) := \frac{1}{2}\int_{\Omega\setminus\overline{\mathcal{O}}} \big(\sigma(u^D) - \sigma(u^N)\big) : \big(\varepsilon(u^D) - \varepsilon(u^N)\big) \qquad \text{where}$$

$$\begin{cases} -\operatorname{div}\sigma(u^D) & = 0 & \text{in } \Omega \setminus \overline{\mathcal{O}}, \\ \sigma(u^D)\, n & = 0 & \text{on } \Gamma, \\ u^D & = f & \text{on } \Upsilon, \end{cases} \qquad \begin{cases} -\operatorname{div}\sigma(u^N) & = 0 & \text{in } \Omega \setminus \overline{\mathcal{O}}, \\ \sigma(u^N)\, n & = 0 & \text{on } \Gamma, \\ \sigma(u^N)\, n_\Upsilon \cdot n_\Upsilon & = g & \text{on } \Upsilon, \\ u^N \cdot \tau & = f \cdot \tau & \text{on } \Upsilon. \end{cases}$$

## 2. Numerical results

We focus on the case of a mechanical structure with four cavities, having the same radius $R_{\text{exact}} = 0.3$ and centered at $(-1.65; 0)$, $(1.65; 0)$, $(0; -1.65)$ and $(0; 1.65)$, to be recovered. Using the topological gradient method [1], one can obtain the following numerical result. We remark that the negative level lines of the topological gradient presented in Figure 1 coincide with the actual cavities.

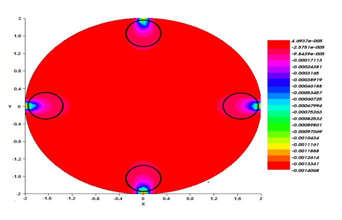

Figure 1. Superposition of the actual cavities and negative level lines of the topological gradient

## References

[1] M. Abdelwahed, M. Hassine, *Topological optimization method for a geometric control problem in Stokes flow*, Appl. Numer. Math. (2009), 59(8), pp. 1823–1838.