MAGNETIC MOMENT RECOVERY FROM PARTIAL FIELD MEASUREMENTS

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Abstract

In geosciences and paleomagnetism, estimating the remanent magnetization in old rocks is an important issue in order to study the Earth magnetic field. However, the magnetization can not be directly measured, but it produces an external magnetic field that can be recorded.

We consider the situation of very thin and weakly magnetized rocks samples, therefore modeled as planar sets $S \subset \mathbb{R}^2 \times \{0\}$, of magnetization $\boldsymbol{m}$ (a 3-dimensional vector field). Scanning microscopes furnish measurements $b_3[\boldsymbol{m}]$ (tiny, a few nano Teslas) of the vertical component of the associated magnetic field on a planar region $Q \subset \mathbb{R}^2 \times \{h\}$ located at some fixed height $h > 0$ above the sample plane. We assume that both S and Q are Lipschitz-smooth bounded connected open planar sets and that the magnetization $\boldsymbol{m}$ belongs to $[L^2(S)]^3$, whence $b_3[\boldsymbol{m}] \in L^2(Q)$. Such magnetization possess net moments $\langle \boldsymbol{m} \rangle \in \mathbb{R}^3$ defined to be their integral on S.

Recovering the magnetization $\boldsymbol{m}$ or its net moment $\langle \boldsymbol{m} \rangle$ from the available measurements $b_3[\boldsymbol{m}]$ are inverse problems for Poisson-Laplace partial differential equations in the upper half-space $\mathbb{R}^3_+$ with right hand side in divergence form. Indeed, Maxwell's equations link the divergence of $\boldsymbol{m}$ to the Laplacian of a scalar magnetic potential in $\mathbb{R}^3_+$ whose normal derivative on Q coincides with $b_3[\boldsymbol{m}]$. Note that Neumann type data $b_3[\boldsymbol{m}]$ are available on $Q \subset \mathbb{R}^3_+$, while we aim at recovering $\boldsymbol{m}$ or $\langle \boldsymbol{m} \rangle$ on S. We thus face recovery issues on the boundary of the harmonicity domain from (very partial) data available inside.

These inverse problems are typically ill-posed and require regularization in order to be solved. Indeed, magnetization recovery issues lack uniqueness, due to the existence of silent sources $\boldsymbol{m} \neq 0$ such that $b_3[\boldsymbol{m}] = 0$, see [2]. Although such sources have vanishing net moment $\langle \boldsymbol{m} \rangle$, moment estimation turns out to be unstable with respect to measurements errors.

We will discuss how to regularize and solve these issues by stating them as minimization problems and adding appropriate constraints. We will especially consider moment $\langle \boldsymbol{m} \rangle$ recovery issues from given values of $b_3[\boldsymbol{m}]$, formulated as best norm-constrained approximation problems, see [1], show well-posedness and constructive results.

References

[1] B. Atfeh, L. Baratchart, J. Leblond, J. R. Partington, *Bounded extremal and Cauchy-Laplace problems on the sphere and shell.* J. Fourier Analysis and Applications (2010), 16(2), 177–203.

[2] L. Baratchart, D. Hardin, E. A. Lima, E. B. Saff, B. P. Weiss, *Characterizing kernels of operators related to thin-plate magnetizations via generalizations of Hodge decompositions.* Inverse Problems (2013), 29.