A KALMAN RANK CONDITION FOR THE INDIRECT CONTROLLABILITY OF COUPLED SYSTEMS OF LINEAR OPERATOR GROUPS

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Abstract. We give a necessary and sufficient condition of Kalman type for the indirect controllability of systems of groups of linear operators, under some "regularity and locality" conditions on the control operator which fit very well the case of distributed controls. Moreover, in the case of first order in time systems, when the Kalman rank condition is not satisfied, we characterize exactly the initial conditions that can be controlled.

1. Introduction

We study the exact controllability for some classes of linear coupled systems with less controls than equations. More precisely, we consider a system of m linear equations where on each equation we have the same operators which is supposed to be a generator of a group on a Hilbert space. We manage to control this system using n ($n < m$) control functions thanks to constant coupling terms of zero order. Under some "regularity and locality" conditions on the control operator and under the condition of controllability for the scalar system associated, we obtain a necessary and sufficient algebraic condition of Kalman type for the exact controllability of this system of equations. The method of this proof was first introduced in [8] and called *fictitious control*. It has been then been used in different context notably in [2] and [6]. It can be described as follows: we first control our system of equations with n controls, one on each equation using in particular [[7], Theorem 1.4]. This control is called a *fictitious control* because it is destined to disappear at the end of the reasoning. Then we try to eliminate this control thanks to algebraic manipulations and replace it by a control operator which has a determined algebraic structure and may not act on all equations. For instance we can apply these results to wave and schrodinger operators under a Geometric Optics Condition (GCC in short) (see [4]).

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