SHAPE OPTIMIZATION IN STATIC PERFECT PLASTICITY

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The shape optimization of a structure, $\Omega \subset \mathbb{R}^d$, obeying a static perfect plastic law presents the difficulty of having as state constraint a variational inequality: noting $\partial\Omega = \Gamma_N \cup \Gamma_0$ with $\Gamma_N \cap \Gamma_0 = \emptyset$, find $\sigma \in M \cap K$ and $u \in BD(\Omega)$ [6] such that,

$$(1) \qquad \begin{cases} \int_\Omega C^{-1}\sigma : (\tau - \sigma)\ dx + \int_\Omega u \cdot div\,(\tau - \sigma)\ dx \geq 0, \quad \forall \tau \in M \cap K, \\ \int_\Omega \sigma : e\,(v)\ dx = \int_{\Gamma_N} g \cdot v\, ds + \int_\Omega f \cdot v\, dx, \quad \forall v \in H^1_{\Gamma_0}(\Omega), \end{cases}$$

where σ is the stress tensor, u the displacement, g stands for a force on a part of the surface, f a volume force, $e(v) = \frac{1}{2}\left(\nabla v + {}^t\nabla v\right)$, C the Hooke tensor, $H^1_{\Gamma_0} = \left\{v \in H^1(\Omega)^d | v = 0 \text{ on } \Gamma_0\right\}$, $K = \left\{\tau \in L^2(\Omega)^{d\times d} | \phi(\tau) \leq 0\right\}$ a cone, ϕ the yield function and $M = \left\{\tau \in L^2(\Omega)^{d\times d} | \tau = {}^t\tau,\ \tau n = g \text{ on } \Gamma_N\right\}$. The sensitivity of the solution (σ, u) with respect to the shape, was studied for a particular case in [5], using the results of [3], pointing its non-differentiability.

In order to avoid using subgradient algorithms, it is easier to regularize the system (1). Using the projection on the convex cone K, one can use a Perzyna penalty type ($\frac{1}{\epsilon}|\sigma - P_K(\sigma)|^2$) such as in [2]. Or one can remark that the problem can be rewritten as:

$$(2) \qquad \int_\Omega P_K\big(Ce\,(u)\big) : e\,(v)\ dx = \int_{\Gamma_N} g \cdot v\, ds + \int_\Omega f \cdot v\, dx, \quad \forall v \in H^1_0(\Omega),$$

where P_K is the orthognal projection on K. This formulation is analyzed for instance in [4], and is used to numerically compute the solutions of (1) even if it is possible that (2) admits no solution.

Our goal is to use (2), smoothing the projection P_K for the particular case of the Von Mises criterion in such a way that the problem admits a unique solution and is differentiable with respect to the shape. Then we compute the derivative of objectives, such as the displacement of a group of points, thanks to the adjoint method. Finally, using the levelset method [1], we present some shape optimization results.

Acknowledgements. This work has been supported by the RODIN project (FUI AAP 13)

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