EFFECTIVE IMAGE INPAINTING AND RESTORATION BASED ON COMBINED SECOND- AND FOURTH-ORDER DIFFUSION MODEL

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Abstract. In this work, we consider a simple and effective fourth-order variational model for solving image inpainting and restoration problem. Then, we perform a tractable control of the regularization parameters on posterior way combined with mesh adaptation techniques. An important feature of the proposed method is that the investigated PDEs are easy to discretize and the overall adaptive approach is easy to implement numerically

1. Introduction and problem formulation

We consider a partial differential equation model combining second- and fourth-order differential operators for solving image inpainting and restoration problems [1, 3, 4], with emphasis on the recovery of the curvature as well as of low-order sets (open lines, points). We consider an image, f, defined on a domain Ω, (usually a rectangular domain) with piecewise smooth boundary $\partial\Omega$. Let $D \subset \Omega$ be a damaged subregion in Ω where the information is not available and is to be completed. The proposed model is given by the follwing equation:

$$\begin{cases} \partial_t u + a\Delta_\beta(\Delta_\alpha u) - b\Delta_\beta u + \lambda_D(u - f) = 0, & \text{in } \mathbb{R}_+ \times \Omega, \\ \frac{\partial u}{\partial n} = \frac{\partial \Delta_\alpha u}{\partial n} = 0, & \text{on } \mathbb{R}_+ \times \partial\Omega, \\ u(0, x) = f, & \text{in } \Omega, \end{cases} \tag{1.1}$$

where $a, b > 0$ are two weighting constant parameters, $\Delta_\beta u = \mathrm{div}(\beta(x)\nabla u)$. The function λ_D is a Lagrange multiplier and it is chosen such that $\lambda_D = \lambda_0 >> 0$ in the image restoration problem, while in the image inpainting problem, it is chosen such that $\lambda_D = \lambda_0 \chi_{\Omega\setminus D}$ where $\chi_{\Omega\setminus D}$ is the characteristic function of the domain $\Omega \setminus \overline{D}$. Model (1.1) might be considered as a simplified version of the Euler's elastica model [2], where the curvature and the length terms are replaced by the fourth- and second-order derivatives, respectively. The parameters a and b are used to control the trade off between the length and curvature in analogy with Euler's elastica model. Then, we introduce a multi-scale approach, rendered by a variable regularization parameters selected locally, adaptively and in a posterior way. The adaptation allows to control the diffusion coefficients in the reconstruction operator with the aim of obtaining as far as possible fine features of the initial image, e.g., (corners, edges, etc).

2. Numerical examples

In order to test our approach and to compare with the existing methods, we present several numerical examples, in image inpanting and restoration problems, which show the good quality in the recovery of low dimensional sets (edges, corners) and curvature in the inpainted zone.

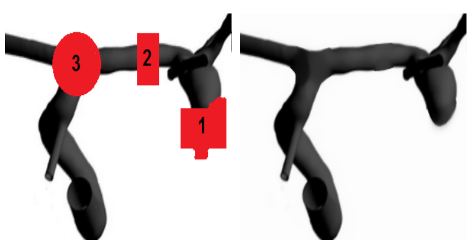

Figure 1. Blood vessel inpainting: Damaged and restored images, respectively.

Figure 2. Connectivity across large gaps: Damaged and restored images, respectively.

References

[1] M. Burger and L. He and C.-B Schönlieb, *Cahn-Hilliard inpainting and a generalization for grayvalue Images*, SIAM J. Imaging Sci., **2(4)** 2009, 1131–1167.
[2] T. Chan, S-H. Kang and J. Shen, *Euler's elastica and curvature based inpainting*, SIAM Journal on Applied Mathematics, **63(2)** 2002, 564–592.
[3] G. Dal Maso, I. Fonseca, G. Leoni, and M. Morini, *A higher order model for image restoration: the one dimensional case*, SIAM Journal on Mathematical Analysis, **40(6)** 2009, 2351–2391.
[4] S. Masnou, *Disocclusion: A variational approach using level lines*, IEEE Trans. Image Process., **11** (2002), 68–76.