NUMERICAL SIMULATION FOR DATA COMPLETION IN ECG INVERSE PROBLEM

R. ABOULAICH [1], E. EL GUARMAH [1,2], N. ZEMZEMI [3].

[1] *Mohammed V University of Rabat, Mohammadia school of Engineering LERMA and LIRIMA Laboratories. Av. Ibn Sina Agdal, Rabat Morocco.*
[2] *Royal Air School, Informatics and Mathematics Departement, DFST, BEFRA, POB 40002, Marrakech, Morocco.*
[3] *INRIA Bordeaux Sud-Ouest Carmen Project, 200 rue de la vieille tour 33405 Talence Cedex, France.*

Abstract. We present in this work, a 3D numerical study with the development of an algorithm for the non-invasive electrocardiography imaging inverse problem. Our goal is to recover the electrical potential on the heart boundary from electrical recordings on the body surface and geometrical informations about the torso. The problem could be considered as a data completion for the diffusion equation where we have both Dirichlet and Neumann boundary conditions on the body surface and we look for the electrical data on the heart surface.

1. Introduction

This problem is known to be ill-posed in the Hadamard sense. The solution does not depend continuously on the data, small errors in the measurement of the torso potentials or thorax geometry could lead to unbounded errors in the solution. The elliptic Cauchy problem used for the numerical simulation is given by:

$$\left\{\begin{array}{lcll} \nabla.\sigma\nabla u & = & 0 & in\ \Omega \\ k\nabla\, u.n & = & \Phi & on\ \Gamma_c, \\ u & = & f & on\ \ \Gamma_c, \end{array}\right. \tag{1.1}$$

where Ω is a bounded domain in $\mathbb{R}^d$ $(d = 2, 3)$ with smooth boundary $\partial\Omega$, which has two open connected components Γ_c and Γ_i. σ is a conductivity parameter, $\Phi \in L^2(\Gamma_c)$ and $f \in L^2(\Gamma_c)$ are given function. The considered Cauchy problem is reformulated as an optimization problem using an optimal control formulation. We use a conjugate gradient optimization procedure in order to solve the optimization problem.We perform different numerical simulations on a 3D realistic geometry with and without noise on the measurements. The first results are very promising.

Keywords: Cauchy problem; Data completion; optimal control formulation; ElectroCardioGraphy Imaging(ECGI).

References

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