DIAGONALITY MEASURES OF HERMITIAN POSITIVE-DEFINITE MATRICES AND APPLICATIONS

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Abstract. In this paper we introduce some measures of diagonality of Hermitian positive-definite matrices. Closed-form expressions of these diagonality measures and discussions of their properties are given in depth. Some problems that occur in the field of signal processing can be formulated as a joint diagonalization problem. Based on these diagonality measures we present some computations of the joint approximate diagonalizer and give an application to the problem of blind carrier frequency offset estimation.

1. Introduction and problem statement

The problem of approximate joint diagonalization (AJD) of a collection of Hermitian positive-definite matrices, plays a vital role in the solution of source separation problems. In signal processing and statistics the problem of AJD has attracted a lot of attention recently and led to impressive results for blind source separation [1] and common principal component problems. In this work, we present different diagonality measures of Hermitian positive-definite matrices and discuss their properties. Then we use them to define cost functions $\mathcal{J}(.)$ of optimization problems for the approximate joint diagonalization of a set of Hermitian positive-definite matrices. For the minimization of these functions, we use the Newton [2] and the steepest descent algorithms, which seek to find a matrix $\mathbf{C}$ that approximately joint diagonalizes the set $\{\mathbf{M}_k\}_{k=1}^{K}$ of Hermitian positive-definite matrices. To use these methods we need to find explicit expressions for both the gradient and Hessian of the cost function in question.

2. Numerical results

Preliminary numerical computations have shown that the Newton method is more efficient than the steepest descent method in terms of number of iterations and CPU time. Application to the carrier frequency offset estimation problem [3] for orthogonal frequency-division multiplexing based systems is considered.

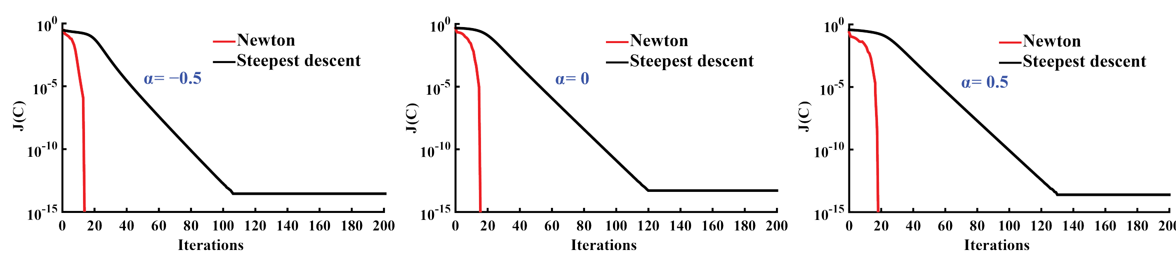

Figure 1. *Plots of $\mathcal{J}(\mathbf{C})$ as a function of the iteration number using the Newton and the steepest descent algorithms for different values of $\boldsymbol{\alpha}$, which parametrizes the family of cost functions.*

References

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