COUPLING PARAREAL WITH NON-OVERLAPPING DOMAIN DECOMPOSITION METHODS

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Abstract. In this work, we present a new parallel algorithm for evolution problems based on coupling parareal with non-overlapping domain decomposition methods in order to perform the parallelism both in time and space. The methods of domain decomposition used are: the Schur complement and the FETI. Numerical experiments illustrate the performance and the scalability of the new method.

1. Introduction

The non-overlapping domain decomposition methods (DDM) are powerful technique for solving elliptic partial differential equations. These methods consist of splitting the global domain into serval non overlapping subdomains and using a parallel solver to solve the local problems. This procedure leads to iterative methods to find interface unknowns connecting domains together, namely the interface displacements in the Schur complement methods. The performance of the method depends on the choice of efficient preconditioner for the interface operators. In this context we use the FETI method. The basic idea of this method is to use Lagrange multipliers to enforce the continuity of the displacement field on the interfaces between the subdomains. Non-overlapping domain decomposition method can be extended to time dependent problems. The numerical temporal evolution schemes are sequential in nature and have thus been viewed as providing a limited means of performance gains. Recent interest arose in order to exploit modern engineering, this development have lead to an increased focus on parallelization with respect to the time variable, for this purpose we adopt the parareal algorithm [1]. This algorithm can be interpreted as a mutltigrid-in-time algorithm in which two levels of grids are present, namely a fine and a coarse grids in time. For problems with a larger size, or larger complexity, the parareal method suffers from the size of the spatial subproblem to solve, in this case we propose to combine the parareal algorithm with the domain decomposition methods.

2. Numerical results

We present the numerical scalability of the parallel in space and time for various number of subdomains $N_s * N_t$, we deduce that DDM and parareal are much less sensitive by the increase of the number of subdomains. We employ a GMRES algorithm for determining the values of interface conditions for DDM.

N_s, N_t	FETI **# GMRES iterations**	Schur Complement **# GMRES iterations**	Parareal **# iterations**
$4, 10$	2	2	5
$8, 20$	3	3	5
$16, 40$	3	3	4
$32, 80$	3	3	4

References

[1] J. Lions , Y. Maday, G. Turinici, *Résolution d'EDP par un schéma en temps pararéel,* C. R. Acad. Sci. Paris (2001)