TOPOLOGICAL GRADIENT APPROACH TO SPECKLE NOISE REMOVING AND EDGE DETECTING IN ULTRASOUND IMAGES

HAMDI HOUICHET [*], MAHER MOAKHER [*], BADREDDINE RJAIBI [*].

[*] *ENIT-LAMSIN, University of Tunis El Manar, B.P. 37, 1002, Tunis-Belvédère, Tunisia.*

Abstract. This paper presents a new approach for restoration and edges detection in ultrasound images corrupted with Rayleigh distributed multiplicative noise (speckle). We consider a semilinear PDE from which we derive the asymptotic expansion of the energy functional. We will give a theoretical expansion of the topological sensitivity approach to detect and preserve set of edges of the ultrasound image during the restoration process. We present numerical results which illustrate the efficiency of the proposed approach.

1. Introduction and problem formulation

In this work, we are concerned with the use of topological gradient method to perform edge detection and image restoration for ultrasound images. This gradient gives the sensitivity of the cost function when a small hole is created at a point. The principle is as follows: we consider Ω a domain of $\mathbb{R}^2$ and $j(\Omega) = \mathcal{J}(u_0)$ a cost function to be minimized, where u_0 is the solution to a given PDE. For a small $\rho > 0$, let Ω_ρ the perturbed domain of Ω by removing an ellipse $E_\rho = x_0 + \rho E$ of size ρ centered at x_0, i.e $\Omega_\rho = \Omega \setminus E_\rho$, where $E = \{(a\cos(\theta), b\sin(\theta)), 0 \leq \theta \leq 2\pi\}$. We can generally prove that the variation of the criterion has the asymptotic expansion:

$$\mathcal{J}(u_\rho) - \mathcal{J}(u_0) = \rho^2 \mathcal{G}(x_0) + o(\rho^2).$$

This expansion is called the topological asymptotic. To minimize the criterion $\mathcal{J}(.)$, we have to create an ellipse where $\mathcal{G}(x_0)$ (the topological gradient at the point x_0) is negative. In order to identify the edges of an image and smooth it elsewhere, we compute the asymptotic expansion of the following cost function $\mathcal{J}(u_\rho) = \frac{1}{2}\int_{\Omega_\rho} |\nabla u_\rho|^2 \mathrm{d}x$, where u_ρ is a solution of the equation:

$$\begin{cases} -\mathrm{div}(\lambda \nabla u_\rho) + \dfrac{u_\rho^2 - f^2}{u_\rho^2} & = & 0 \text{ in } \Omega_\rho, \\ \partial_n u_\rho & = & 0 \text{ on } \Gamma, \\ \partial_n u_\rho & = & 0 \text{ on } \partial E_\rho, \end{cases}$$

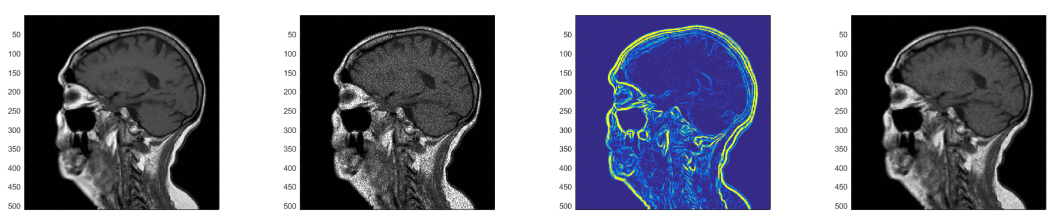
where $\Gamma \cup \partial E_\rho = \partial \Omega_\rho$, λ is a positive constant and f is the noisy image.
To restore the image u we solve the previous equation with the piecewise constant function λ:

$$\lambda = \begin{cases} \lambda_1 & \text{if } x_0 \in \{x \in \Omega \text{ such that } \mathcal{G}(x_0) < \alpha < 0\}, \\ \lambda_0 & \text{otherwise } (\lambda_1 \ll \lambda_0), \end{cases}$$

where α is a given negative threshold.

2. Numerical results

The following figure shows the original brain image (a) perturbed by speckle noise (b) of standard deviation $\sigma = 0.28$, the edges obtained by the topological gradient (c) with $\alpha = -150$ and the recovered image (d) with $\lambda_0 = 2$.



(a) (b) (c) (d)

Figure 1. (a) Original image, (b) noisy image ($PSNR = 22.24$dB, $SNR = 11.61$dB, $SSIM = 0.67$), (c) contour and (d) the restored image ($PSNR = 30.5$dB, $SNR = 19.5$dB, $SSIM = 0.9$).

References

[1] S. Amstutz, *Aspects théoriques et numériques en optimisation de forme topologique*, Thèse de doctorat. Institut national des sciences appliquées de Toulouse (2003).

[2] A. Drogoul and G. Aubert, *The topological gradient method for semi-linear problems and application to edge detection and noise removal*, To appear in Inverse Problem and Imaging (IPI) (2015).