DETECTION AND LOCATION OF A POLLUTION POINT SOURCE WITH DELAYED OBSERVATIONS

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Abstract. We are interested in an inverse problem of recovering the position of a pollutant or contaminant source in a stream water. Advection, dispersive transport and the reaction of the solute is commonly modeled by a linear parabolic equation. Streams of average importance can not be equipped with such monitoring systems and with measure, for cost reasons. It happens when measures are decided by the authorities concerned and carried out after a pollution is noticed, as from certain moment after an accidental spill of pollutants. The observations are thus made with a time delay. In this case, we prouved an identifiability result.

1. Introduction

The biochemical oxygen demand (BOD) is the amount of dissolved oxygen consumed by aerobic bilogical organisms to oxide organic material present in water. The BOD concentration is designed by b. Throughout, we denote $I = \mathbb{R}$ the curvilinear representation of the stream-water and $T > 0$ is a final instant. We use the symbol x for the curvilinear abscissa and t stands for the time variable. The contaminant concentration $b(\cdot,\cdot)$ is governed by the following transport equation

$$\begin{aligned} \partial_t b - (Db')' + (Vb)' + Rb = f(t)\delta_{x-s(t)} \qquad & \text{in } I \times \mathbb{R}, \\ b(\cdot, 0) = b_0 \qquad & \text{in } I, \\ Db'(0,\cdot) = \psi_0, \qquad Db'(L,\cdot) = \psi_L \qquad & \text{in } (0,T). \end{aligned}$$

The longitudinal dispersion coefficient D, the average flow velocity V and the reaction rate R are all in $L^\infty(I)$. The initial condition b_0 and the boundary conditions ψ are known. The inverse problem of source detection mostly dealt with in the specialized literature (see [2]) consists in the determination of $F = (f(\cdot), s(\cdot))$ from some given observations on $b(\cdot,\cdot)$. To make diagnostic statements about a possible contamination, assume two sensors placed at the points 0 and L at the moment $t_m > 0$ such time after the spill of a pollutant. The observation operator is defined by

$$B[F](\cdot) = (b(0,\cdot), b(L,\cdot)).$$

Consider that the measurement functions $(h_0(\cdot), h_L(\cdot))$ are known. Are they significant and sufficient to discriminate the source F. This is actually related to the uniqueness for the inverse problem: *find F satisfying*

$$B[F](t) = (h_0(t), h_L(t)), \qquad \forall t \in (t_m, T).$$

Thanks to the unique continuation theorem (see [1]) and some mathematical techniques, we proved the identifiability result.

References

[1] Saut, J-C. and Scheurer, B., *Unique Continuation for Some Evolution Equations*, Journal of Differential Equations (1987), pp. 118-139.

[2] Andrle, M. and Ben Belgacem, F. and El Badia, A., *Identification of Moving Pointwise Sources in an Advection-Dispersion-Reaction Equation*, Inverse Problems (2011), pp. 025007.