STABILITY RESULTS FOR THE PARAMETER IDENTIFICATION INVERSE PROBLEM IN CARDIAC ELECTROPHYSIOLOGY

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The aim of this work is to establish stability estimates for the parameter identification problem in cardiac electrophysiology modeling. The propagation of the electrical wave in the heart is described by the monodomain equation. The model consists of a reaction-diffusion non linear equation coupled to an ODE system representing the electrical activity of the cell membrane.

$$\left\{\begin{array}{ll} \chi_m \dfrac{\partial V_m}{\partial t} - div(\sigma \nabla V_m) = I_{app} - I_{ion}(V_m, w) & \text{in } \Omega \times (0,T) \\ \dfrac{\partial w}{\partial t} + G(V_m, w) = 0 & \text{in } \Omega \times (0,T) \\ \sigma \nabla V_m . n = 0 & \text{on } \Sigma. \end{array}\right. \tag{0.1}$$

where Ω and Σ denote respectively the domain and the boundary of the heart. The time domain is given by $[0, T]$ and χ_m the membrane capacitance per area unit. The variable V_m denote the action potential, and σ is the bulk conductivity. The term I_{app} is a given external current stimulus, w represents the concentrations of different chemical species, and variables representing the openings or closures of some gates of the ionic channels. And the ionic current I_{ion} and the function $G(V_m, w)$ depends on the considered ionic model. In this study, the dynamics of w and I_{ion} are described by the Mitchell and Schaeffer model :

$$I_{ion}(v, w) = \frac{w}{\tau_{in}} v^2(v-1) - \frac{v}{\tau_{out}} \quad and \quad G(v, w) = \left\{\begin{array}{ll} \dfrac{w-1}{\tau_{open}} & \text{si } v \le v_{gate} \\ \dfrac{w}{\tau_{close}} & \text{si } v > v_{gate} \end{array}\right.$$

The time constants τ_{in}, τ_{out} are respectively related to the length of the depolarization and repolarization. The parameters τ_{open} and τ_{close} are the characteristic times of gate opening and closing respectively and v_{gate} corresponds to the change-over voltage.

In this work we are interested in the identification of the parameter τ_{in} to which the solution is known to be very sensitive. Let τ_{in} and $\widetilde{\tau}_{in}$ be two different strictly positive parameters. We define $f(V_m, w) = I_{app} - I_{ion}(V_m, w)$, $a = \dfrac{1}{\tau_{in}}$, $\widetilde{a} = \dfrac{1}{\widetilde{\tau}_{in}}$ $(V_m, w) = (V_m(a), w(a))$ and $(\widetilde{V}_m, \widetilde{w}) = (V_m(\widetilde{a}), w(\widetilde{a}))$. Let's also denote

$$V = V_m - \widetilde{V}_m \qquad \text{and} \qquad W = w - \widetilde{w}. \tag{0.2}$$

Let ω_0 be an arbitrary non-empty subdomain of Ω, we prove the following stability result

Theorem 1. *Let (V, W) defined as in (0.2) with the initial condition $V_0 \in H^2(\Omega)$.*
Let us assume that

$$\frac{\partial f}{\partial a}(\tilde{V}_m, \tilde{w}) \ge r_0 > 0 \quad for\ some \quad T' > 0 \quad and \quad \forall x \in \Omega\,.$$

Then there exists $C > 0$ such that

$$|a - \widetilde{a}| \le C N_{T', \omega_0}(V, W),$$

where

$$N_{T',\omega_0}(V, W) = \|V\|_{H^1(0,T;L^2(\omega_0))} + \|V\|^2_{L^4(0,T;L^4(\omega_0))} + \|V(T')\|_{H^2(\Omega)} + \|V(T')\|^3_{L^6(\Omega)} + \|W(T')\|_{L^2(\Omega)}.$$