SHAPE OPTIMIZATION FOR INCOMPRESSIBLE LAMINAR FLOWS

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Abstract. In the context of incompressible laminar flows governed by the (Navier-)Stokes equations, we numerically investigate the problem of determining the optimal shape of a duct with several inlets/outlets. This problem is modeled in terms of minimization/maximization of dissipated energy or vorticity functionals, under the aforementioned geometrical constraints. We introduce a Lagrangian type algorithm where steps are refined to deal with the complexity of such a problem.

1. Introduction

We investigate the following problem

$$\min_{\Omega \in \mathcal{O}_{ad}, G(\Omega)=0} J(\Omega, u_\Omega)$$

where J depends on both Ω and the solution u_Ω of (Navier-)Stokes equations. $\mathcal{O}_{ad}$ is associated to the required regularity and box constraints, while G is associated to geometrical constraints. The present work has applications in biomedicine, namely for the optimal design of vascular bypasses.

2. Numerical results

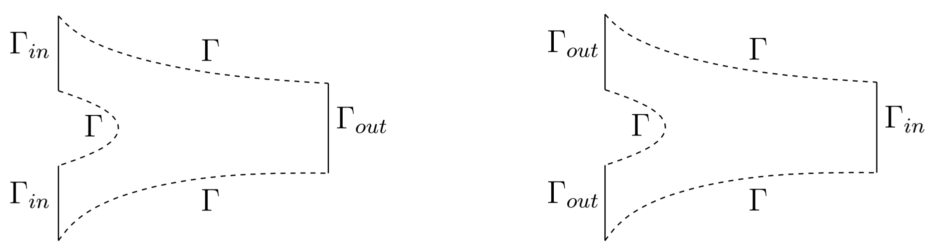

Figure 1. Left : configuration (I) ; right : configuration (II). Γ_{in} ($\mathbf{u} = \mathbf{u}_0$) and Γ_{out} ($\sigma(\mathbf{u}, p)\mathbf{n} = -p_0\mathbf{n}$) are allowed to move tangentially, Γ is allowed to move normally.

In the case $G(\Omega) = |\Omega| - V_0$, we identify several behaviors of the minimizing sequences, depending both on the physical model and the boundary conditions. For instance we either observe the closing of branches thus suggesting a non-existence phenomenon, or the convergence of the algorithm to a dyadic tree shape.

References

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