MAGNETIZATION FEATURES RECOVERY BASED ON KELVIN TRANSFORMATION AND FOURIER ANALYSIS

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Abstract

Some ancient rocks and meteorites possess remanent magnetization and thus store valuable record of the past magnetic field on Earth and other planets or satellites. Deducing magnetization of a geosample hinges on processing the data of the weak magnetic field available in its nearest neighborhood from SQUID microscope measurements.

Vertical component of the magnetic field produced by the sample is typically measured on a part of the horizontal plane at height $x_3 = h > 0$ where it equals [1, 2]

$$B_3(\mathbf{x}, h) = \frac{1}{4\pi} \iint_Q \frac{3h\left[M_1(\mathbf{t})(x_1 - t_1) + M_2(\mathbf{t})(x_2 - t_2)\right] + M_3(\mathbf{t})\left[2h^2 - (x_1 - t_1)^2 - (x_2 - t_2)^2\right]}{\left[(x_1 - t_1)^2 + (x_2 - t_2)^2 + h^2\right]^{5/2}} dt_1 dt_2,$$

whereas the magnetization of the sample is an unknown function $\mathbf{M}(\mathbf{x}) = (M_1(\mathbf{x}), M_2(\mathbf{x}), M_3(\mathbf{x}))^T$, $\mathbf{x} := (x_1, x_2)^T$ supported on a subset Q of the horizontal plane at $x_3 = 0$.

The goal of this work is to obtain explicit formulas for characterization of the magnetic sample in whole. Namely, given the measured field $B_3(\mathbf{x}, h)$ on a part of the plane which we take to be a disk D_A of radius A, we want to deduce first few algebraic moments of the magnetic sample. Among those, the components of the net moment $\mathbf{m} = (m_1, m_2, m_3)^T := \iint_Q \mathbf{M}(\mathbf{x})\, dx_1 dx_2$ is especially important as it contains the most valuable information about the sample from physical point of view. It is known [3] that the net moment of the source can be obtained from knowledge of the potential or the field on a sphere surrounding the sources by means of integration of the data against first spherical harmonics. First, we demonstrate to which extent and how one can adapt such spherical geometry methods for the planar case in question. Namely, we obtain results performing mapping to the sphere by means of Kelvin transform and evaluate specific asymptotic projections onto few spherical harmonics. Then, we study the original problem in Fourier domain and devise a systematic way of extraction of moment information by analysing wave vectors of different smallness in magnitude. The latter technique yields some of the formulas obtained in Kelvin domain as well as their improved analogs. In particular, we deduced the following asymptotic formulas for the net moment components

$$\begin{aligned} m_i &= 2 \iint_{D_A} \left(1 + \frac{4x_i^2}{3A^2}\right) x_i B_3(\mathbf{x}, h)\, dx_1 dx_2 + \mathcal{O}\left(\frac{1}{A^2}\right), \quad i = 1, 2, \\ m_3 &= 2A \iint_{D_A} B_3(\mathbf{x}, h)\, dx_1 dx_2 + \mathcal{O}\left(\frac{1}{A^2}\right). \end{aligned}$$

The developed techniques could be extended to other inverse source characterization problems with data available on the plane and harmonic properties of the field.

References

[1] L. Baratchart, D. P. Hardin, E. A. Lima, E. B. Saff, B. P. Weiss, "Characterizing kernels of operators related to thin-plate magnetizations via generalizations of Hodge decompositions", Inverse Problems, 29 (1), 2013.

[2] E. A. Lima, B. P. Weiss, L. Baratchart, D. P. Hardin, E. B. Saff, "Fast inversion of magnetic field maps of unidirectional planar geological magnetization", J. Geophys. Res.: Solid Earth, 118 (6), 2723-2752, 2013.

[3] F. J. Lowes, B. Duka, "Magnetic multipole moments (Gauss coefficients) and vector potential given by an arbitrary current distribution", Earth Planets Space, 63,1-6, 2011.