Simultaneous estimation of two parameters in a porous media

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Abstract

We identify simultaneously the hydraulic transmissivity and the storage coefficient in a ground water flow governed by a linear parabolic equation (0.1).

$$\begin{array}{lcll} S\dfrac{\partial \Phi}{\partial t} - \operatorname{div}(T\nabla\Phi) & = & Q & \text{in } \Omega \times (0, t_f) \\ \Phi & = & \Phi_d & \text{on } \Gamma_D \times (0, t_f) \\ (-T\nabla\Phi)\cdot n & = & \Phi_N & \text{on } \Gamma_N \times (0, t_f) \\ \Phi(0) & = & \Phi_0 & \text{in } \Omega \end{array} \tag{0.1}$$

where Ω is a bounded connected domain of $\mathbb{R}^2$, the time variable t belongs to the interval $(0, t_f)$, S is the storage coefficient and T is the hydraulic transmissivity, Φ is the piezometric head and Q is a distributed source terms, n is the outer normal to Ω, Γ_D and Γ_N are a partition of the boundary of Ω denoting respectively Dirichlet and Neumann conditions. Both coefficients are assumed to be space variable functions piecewise constant. Therefore the unknowns are the coefficient values as well as the geometry of the zones where these parameters are constant. The identification problem is formulated as the minimization of a misfit least-square function (0.2).

$$J(S,T) = \frac{1}{2}\sum_{i=1}^{N_t}\sum_{j=1}^{m}(\Phi(S,T)(t_i, M_j) - d_{ij}^{obs})^2 \tag{0.2}$$

where d_{ij}^{obs} is the piezometric head measured at time t_i and at point $M_j(x_j, y_j)$ and $\Phi(S,T)(t_i, M_j)$ is the model output for the current coefficients S and T at the same time and the same point.

Using refinement indicators, we build the parameterization iteratively, going from a one zone parametrization to a parametrization with m zones where m is an optimal value to identify. We distinguish the cases where the two parameters have the same parameterization and different parameterizations.

To improve the resolution of the inverse problem, we incorporate a posteriori error estimations defined to refine the computing mesh in an adaptive parametrization algorithm.

Key-words: Inverse problem, parameter estimation, parameterization, refinement indicators, storage coefficient, hydraulic transmissivity.

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