EXPLICIT A POSTERIORI ERROR ESTIMATES IN A MOMENT METHOD FOR RECOVERING BOUNDARY DATA

B. Achchab, A. Sakat and A. Souissi

Laboratoire de Modlisation et Combinatoire, Safi, Morocco
University Cadi Ayyad Marrakech
a.sakat@uca.ma

ABSTRACT

We consider a data completion method for the Cauchy problem of Laplace equation. For an approximate solution given by any one of algorithms of the data completion methods, we give the moment of the error between the true solution and the approximate one. This error indicator is used to improve the approximate solution, to give a knowledge of stability and can be used as a stopping criterion.

1. MAIN FACTS

Let Ω be a Lipschitz bounded domain in $\mathbb{R}^2$. ν is the normal unit to the boundary $\partial\Omega$, oriented outward. We assume that $\partial\Omega$ is partitioned into two open connected portions Γ_C and Γ_I such that $\partial\Omega = \overline{\Gamma}_I \cup \overline{\Gamma}_C$. Each of them is of a non-vanishing measure. The data completion problem for the Laplace equation is given by :
Find (φ, ψ) on Γ_I such that there exists a temperature field u satisfying

$$\begin{cases} \Delta u = 0 \ \text{ in } \Omega, \\ u = f \ \text{ on } \Gamma_C, \\ \dfrac{\partial u}{\partial \nu} = \phi \ \text{ on } \Gamma_C, \end{cases} \tag{1}$$

with $\varphi = u$ and $\psi = \dfrac{\partial u}{\partial \nu}$ on Γ_I.

Let (u_h, w_h) be an approximation of $(u, \dfrac{\partial u}{\partial \nu})$ on Γ_I, by applying any one of the data completion methods see for example [1, 4], we define two error functions on Γ_I, $e_1 = u|_{\Gamma_I} - u_h$ and $e_2 = \dfrac{\partial u}{\partial \nu}|_{\Gamma_I} - w_h$.

In this work, we propose a new procedure, based on the moment technique [2], to explicitly approximate the error e_1 and the error e_2. More precisely, let $\{v_j, j \in \mathbb{N}\}$ be a sequence of orthonormal functions such that $\Delta v_j = 0,\ \forall j \in$ and $\overline{Span\{v_j|_{\Gamma_i}\}_{j=0}^{\infty}} = L^2(\Gamma_I)$. Assume (1) has a solution u such that $u|_{\Gamma_I} \in L^2(\Gamma_I)$.

Let m_j^k, $j \in \mathbb{N}$, be the moments of e_k, $k = 1, 2$ defined by $m_j^k = \langle e_k, v_j \rangle = \int_{\Gamma_I} e_k v_j ds$, where $\langle ., . \rangle$ is the inner product of $L^2(\Gamma_I)$. By means of the given data and v_j, $j \in \mathbb{N}$, we give m_j^k $k = 1, 2$. The obtained approximation of e_1 and e_2 permits to build an a posteriori error estimation of residual type [3] in order to improve the given approximate solution and to have a stopping criterion. We give some convergence and stability results and we introduce an algorithm for solving the two moment problems. To illustrate the proposed approach, we give some numerical experiments.

2. ACKNOWLEDGMENT

This work was partly supported by VOLUBILIS Project number M.A/13/286, LIRIMA, LIEM2I and Hassan-I University Fund; Hydrinv mediterraneen prject 3+3.

3. REFERENCES

[1] S. Andrieux, T. N. Baranger and A. Ben Abda,*Solving Cauchy problems by minimizing an energy-like functional*, Inverse Problems, vol.22, no. 1, (2006) 115-133.

[2] D. D. Ang, R. Gorenflo, L. K. Vy and D. D. Trong, *Moment Theory and Some Inverse Problems in Potential Theory and Heat Conduction*, Lecture Notes in Mathematics 1792, Springer-Verlag Berlin Heidelberg 2002.

[3] R. Becker and R. Rannacher, *An optimal control approach to a posteriori error estimation in finite element methods*, Acta Numerica (2001) 10, 1-102.

[4] A. Chakib and A. Nachaoui, *Convergence analysis for finite element approximation to an inverse Cauchy problem*, Inverse Problems, vol. 22, no. 4(2006) 1191-1206.